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QUESTION WORKBOOK

# Leibnitz's theorem and the Weierstrass substitution

Further Pure 1 · Year FM

6 questions · 20 marks

PRINT EDITION

**SECTION A** · Warm-up

**A1** State Leibnitz's theorem for the  $n$ th derivative of a product  $fg$ . [2]

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**A2** Under the substitution  $t = \tan(x/2)$ , write down  $\sin x$ ,  $\cos x$  and  $dx$  in terms of  $t$ . [3]

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**SECTION B** · Standard

**B1** Use Leibnitz's theorem to find the fourth derivative of  $y = x^3e^x$ . [4]

**B2** Use  $t = \tan(x/2)$  to evaluate  $\int 1/(1 + \cos x) dx$  from 0 to  $\pi/2$ , exactly. [4]

**B3** Use  $t = \tan(x/2)$  to show that  $\int \sec x \, dx = \ln|(1+t)/(1-t)| + c$ . [4]

### SECTION C · Stretch

**C1** Explain why Leibnitz's theorem is especially efficient when one factor of the product is a polynomial. [3]