inkMaths

ANSWER BOOK

Limits and L'Hospital's rule

Further Pure 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 If f and g both tend to 0, or both tend to infinity, as x approaches a, then the limit of f/g equals the limit of f'/g', provided that second limit exists. The top and bottom are differentiated separately: this is not the quotient rule. [2]

A2 Both parts vanish at 0, so apply the rule: $\sin x/(2x)$, still 0/0. A second application gives $\cos x/2 \to 1/2$. Alternatively $\cos x = 1 - x^2/2 + \ldots$, so the numerator is $x^2/2 + \ldots$ and the quotient tends to 1/2 in one line. [3]

SECTION B · Standard

B1 Series: $e^x = 1 + x + x^2/2 + \ldots$, so the numerator is $x^2/2 + x^3/6 + \ldots$ and the quotient tends to 1/2. L'Hospital: $(e^x - 1)/(2x)$, still 0/0, then $e^x/2 \to 1/2$. Both routes agree; the series route needed no differentiation at all. [4]

B2 The numerator is $x^3/3 + 2x^5/15 + \ldots$, so dividing by x^3 gives $1/3 + 2x^2/15 + \ldots \to 1/3$. L'Hospital would need three rounds and increasingly ugly derivatives of $\sec^2 x$, which is why the series is the sensible tool once one is supplied. [4]

B3 Take logarithms: $\ln y = x \ln(1 + 3/x) = \ln(1 + 3/x)/(1/x)$, which is 0/0 as $x \to \infty$. Applying L'Hospital, or expanding $\ln(1 + u) \approx u - u^2/2$, gives $\ln y \to 3$. So the limit is $e^3 \approx 20.09$. Power forms must be logged before the rule can see a quotient. [4]

SECTION C · Stretch

C1 L'Hospital differentiates numerator and denominator separately: $(1 + \cos x)/(1 + \sec^2 x) \to 2/2 = 1$. The quotient rule computes the derivative of the whole fraction, which answers a different question entirely. The correct limit is 1, which the series route confirms: both parts are $2x + \ldots$ near zero. [3]