inkMaths

ANSWER BOOK

Linear programming: formulation and graphical solution

Decision Mathematics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The variables defined in words with their units, the objective to be maximised or minimised, and the constraints as inequalities. The non-negativity conditions are the commonest omission; they bound the region and carry a mark of their own. [2]

A2 $3x + 2y + s_1 = 20$, where s_1 is a slack variable taking up the unused amount. $5x + 2y - s_2 + t_1 = 30$, where s_2 is a surplus recording the excess over 30 and t_1 is an artificial variable providing a starting solution. [3]

SECTION B · Standard

B1 The vertices are (0, 0), (10, 0), (0, 8) and the intersection of the two lines. Solving $x + y = 10$ with $2x + 3y = 24$: $2x + 3(10 - x) = 24$, so $30 - x = 24$ and $x = 6$, $y = 4$. Evaluating P: 0, 40, 40 and 44. The maximum is 44 at (6, 4), which is nearer the origin than (10, 0). [4]

B2 Draw one line of constant objective value, say $4x + 5y = 20$, and slide it parallel away from the origin. The last corner it touches before leaving the region is the optimum, here (6, 4). It also shows at once when the objective is parallel to a constraint, in which case a whole edge is optimal and there are infinitely many solutions, which testing the vertices would report only as a tie. [4]

B3 At (6, 4): $6 + 4 + s_1 = 10$ gives s_1 = 0, and $12 + 12 + s_2 = 24$ gives s_2 = 0. Both constraints are binding, so there is no spare capacity in either: relaxing either one would allow a better answer. [3]

SECTION C · Stretch

C1 Here (6, 4) is already integral, so the answer is unchanged at 44. In general the corner need not be integral, and rounding may land outside the feasible region or on a worse point: rounding (6.4, 3.8) up to (7, 4) breaks $x + y \le 10$, and down to (6, 3) gives only 39 when (5, 4) gives 40. The safe method is to test all the lattice points in the region near the corner and take the best. [4]