ANSWER BOOK

Lines and planes in three dimensions

Further vectors · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Direction PQ = (2, 1, 3), so r = (2, 1, 0) + λ(2, 1, 3). Any point of the line and any non-zero multiple of the direction give an equally correct equation. [2]

A2 From x: 2 + 2λ = 8, so λ = 3. Then y = 1 + 3 = 4 and z = 0 + 9 = 9, both matching. All three coordinates agree at the same λ, so the point is on the line. [3]

SECTION B · Standard

B1 Cartesian: 2x - y + 2z = 9. At the point: 6 - 1 + 4 = 9, so yes, it lies in the plane. The scalar product form and the cartesian form are the same statement in different notation. [3]

B2 In-plane directions: AB = (1, 2, 0) and AC = (2, 0, 3). A normal (a, b, c) needs a + 2b = 0 and 2a + 3c = 0; choosing b = 1 gives a = -2, c = 4/3, and scaling by 3 gives n = (-6, 3, 4). Then d = n·A = 1, so -6x + 3y + 4z = 1. Checking B and C: -12 + 9 + 4 = 1 and -18 + 3 + 16 = 1. [5]

B3 Solve each coordinate for λ: (x - 2)/2 = (y - 1)/1 = z/3. Each fraction equals λ, so setting them equal states that one value of the parameter produces all three coordinates at once, which is exactly membership of the line. [4]

SECTION C · Stretch

C1 The x-component of the direction is zero, so (x - a)/0 is meaningless: x never changes. Write x = a separately, with the remaining fractions equal: x = a, (y - b)/2 = (z - c)/5. A zero denominator signals a frozen coordinate, not a formula to force. [3]