InkMaths

QUESTION WORKBOOK

Locating roots and iteration

Numerical methods · Year 13

8 questions · 20 marks

NAVY SCREEN EDITION

SECTION A · Warm-up

A1 $f(x) = x^3 + x - 5$. Show that $f(x) = 0$ has a root between $x = 1$ and $x = 2$. [2]

A2 Explain why a change of sign of f between a and b does not always mean a root lies in (a, b), and give the condition that repairs the argument. [2]

A3 Show that $x^3 + x - 5 = 0$ can be rearranged into the iterative form $x = (5 - x)^{1/3}$. [2]

SECTION B · Standard

B1 Using $x_{n+1} = (5 - x_n)^{1/3}$ with $x_0 = 1.5$, find x_1, x_2 and x_3 to 5 decimal places. [3]

B2 Show that $\alpha = 1.516$ is a root of $x^3 + x - 5 = 0$ correct to 3 decimal places. [3]

B3 The cobweb diagram for this iteration spirals inwards towards the root. State the feature of $g(x) = (5 - x)^{1/3}$ near the root that guarantees convergence. [2]

SECTION C · Stretch

C1 The same equation rearranges to $x = 5 - x^3$. Starting from $x_0 = 1.5$, calculate x_1, and explain what happens to this iteration. [3]

C2 An iteration converges to a value α. Explain why α must satisfy the original equation, and why the sequence displayed on a calculator appearing to settle is not, by itself, a proof of accuracy. [3]