InkMaths

ANSWER BOOK

Log graphs and exponential models

Exponentials and logarithms · Year 12

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Plot log y against log x. If the law holds the points fall on a straight line with gradient n and intercept log a, since log y = log a + n log x. Straightness on logged axes is the evidence. [2]

A2 Plot log y against x itself, not log x: the law gives log y = log a + x log b, a line with gradient log b and intercept log a. Which axis gets logged is the only difference between the two tests. [2]

SECTION B · Standard

B1 The log-log gradient is n = (log 135 - log 40)/(log 9 - log 4) = log 3.375 ÷ log 2.25 = 1.5 exactly, since $3.375 = 2.25^{1.5}$. Then $a = 40/4^{1.5} = 40/8 = 5$, and the law is $y = 5x^{1.5}$. The second point checks it: $5 \times 9^{1.5} = 5 \times 27 = 135$. [4]

B2 The initial mass is the front constant, 60 g. After 5 days, $m = 60e^{-0.5} = 36.4$ g. Halving means $e^{-0.1t} = ½$, so t = ln 2/0.1 = 6.93 days. The half-life calculation is the logs machinery working inside the model. [3]

B3 Plot log y against x. Taking logs: log y = log 3 + x log 5, so the gradient is log 5 = 0.699 and the intercept is log 3 = 0.477. The growth factor lives in the gradient, the starting value in the intercept. [3]

SECTION C · Stretch

C1 The line says log y = 0.5 log x + log 6, so the law is $y = 6x^{0.5} = 6\sqrt{x}$. At x = 25, y = 6 × 5 = 30. Undoing the logs turns the intercept back into the front constant and the gradient back into the power. [4]

C2 $m(t + 7)/m(t) = e^{-0.1(t+7)}/e^{-0.1t} = e^{-0.7} = 0.497$, with the t-dependence cancelling entirely. Equal time steps multiply the mass by equal factors: the signature no other kind of curve shares. [2]