InkMaths

ANSWER BOOK

Logarithms and their laws

Exponentials and logarithms · Year 12

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 A logarithm is an exponent: $2^5 = 32$ gives 5, $5^3 = 125$ gives 3, and $10^{-2} = 0.01$ gives -2. The negative answer trips people; logs of numbers below 1 are negative, not impossible. [2]

A2 Combine left to right: $\log 8 + \log 5 = \log 40$, and $\log 40 - \log 10 = \log (40/10) = \log 4$. Addition multiplies, subtraction divides: the index laws wearing log clothing. [2]

SECTION B · Standard

B1 The multiple moves inside first: $\frac{1}{2} \log 49 = \log \sqrt{49} = \log 7$. Then $\log 7 + \log 2 = \log 14$. The fractional power law is the same third law that handles whole numbers; a half becomes a square root. [3]

B2 Take ln of both sides: $2x - 1 = \ln 12$. So $x = (\ln 12 + 1)/2 = 1.742$ to 4 significant figures. The log peels the exponential off the left side; everything after that is linear algebra. [3]

B3 Apply e to both sides: $2x + 5 = e^3$. So $x = (e^3 - 5)/2 = 7.543$ to 4 significant figures. This equation needed the exponential applied rather than peeled off: deciding which way round is the real skill. [3]

B4 The bases refuse to match, so take logs of both sides: $(2x + 1) \log 5 = \log 40$. Then $2x + 1 = \log 40/\log 5 = 2.292$, and $x = 0.6460$ to 4 significant figures. Any base of logarithm works, because the base cancels in the ratio. [4]

SECTION C · Stretch

C1 Let $u = 2^x$; since $2^{2x} = u^2$, the equation is $u^2 - 5u + 4 = 0$, which factorises as $(u - 1)(u - 4) = 0$. So $2^x = 1$, giving $x = 0$, or $2^x = 4$, giving $x = 2$. A quadratic wearing an exponential disguise: the substitution unlocks it, and both roots here convert back cleanly. [4]