inkMaths

ANSWER BOOK

Maclaurin series

Further algebra and series · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $f(x) = f(0) + f'(0)x + f''(0)x^2/2! + f'''(0)x^3/3! + \ldots$: the coefficient of x^r is the rth derivative at zero divided by r factorial. [2]

A2 The derivatives at 0 cycle 1, 0, −1, 0, 1: so $\cos x = 1 - x^2/2! + x^4/4! + \ldots = 1 - x^2/2 + x^4/24 + \ldots$. Only even powers survive, matching cos being an even function. [3]

SECTION B · Standard

B1 $1 - 0.02 + 0.0000667 = 0.980067$, and the true value is 0.980067 to six decimal places. The next term is of order $0.2^6/720$, far below the sixth decimal, which is why the match is so tight. [3]

B2 Substitute 3x into the exponential series: $1 + 3x + 9x^2/2 + 27x^3/6 = 1 + 3x + (9/2)x^2 + (9/2)x^3 + \ldots$. The x^3 coefficient is $27/3! = 9/2$. Substitution into a standard series beats fresh differentiation every time it is available. [4]

B3 ln(1 + x) converges for −1 < x ≤ 1. Substituting 3x means the substituted quantity must stay in that window: −1 < 3x ≤ 1, so −1/3 < x ≤ 1/3. The window transforms with the same substitution as the series. [4]

SECTION C · Stretch

C1 Subtracting the two log series cancels the even powers and doubles the odd ones: $2(x + x^3/3 + x^5/5 + \ldots)$. At $x = 1/3$ the argument is $(4/3)/(2/3) = 2$, and two terms give $2(1/3 + 1/81) = 0.6914$, against $\ln 2 = 0.6931$. Far faster than the plain $\ln(1 + x)$ series at $x = 1$, which needs hundreds of terms for the same accuracy. [4]