InkMaths

ANSWER BOOK

Maximum flow and the labelling procedure

Decision Mathematics 2 · Year FM

6 questions · 21 marks

NAVY EDITION

SECTION A · Warm-up

A1 One is the spare capacity, written as an arrow in the direction of the arc: how much more can be pushed forward. The other is the flow already there, written as an arrow against the arc: how much could be taken back. Both are needed, because an augmenting path may reduce the flow on an arc as well as increase it. [2]

A2 It carries 4 units, the smallest spare capacity along the path. The arc that had spare capacity 4 becomes saturated, so no later path can use it in the forward direction. [2]

SECTION B · Standard

B1 S-A-C-T has spare capacities 12, 7 and 10, so push 7 and saturate AC.
S-A-D-T now has 5, 6 and 11 available, so push 5, which uses up SA.
S-B-D-T has 9, 8 and 6 left, so push 6, saturating DT.
Total flow = 7 + 5 + 6 = 18. The arcs carry SA 12, SB 6, AC 7, AD 5, BD 6, CT 7, DT 11.
No augmenting path remains: from S only SB has spare capacity, from B only BD, and D's only outlet DT is full. [6]

B2 Take the source side to be {S, A, B, D}, everything still reachable from S by an augmenting step. The arcs from that set to {C, T} are AC and DT, of capacities 7 and 11, so the cut has capacity 18. Both are carrying their full capacity.
A flow of 18 and a cut of 18 agree, so by the max-flow min-cut theorem the flow is maximal and this cut is minimal. [4]

B3 An augmenting path may run backwards along an arc that already carries flow, reducing it there so that more can travel by another route. So a path can still exist even when every direct route is blocked by a full arc. Only the absence of any flow-augmenting path, forwards or backwards, settles the matter, and the cut of matching capacity is what proves it. [3]

SECTION C · Stretch

C1 A lower capacity is handled by starting with 4 units already on BD, so its labels open at spare capacity 4 and potential backflow 4, and the backflow is never allowed to drop below the required minimum. The path S-B-D-T supplies those 4 units at the outset.
The flow of 18 already sends 6 along BD, which is more than the required 4, so the constraint is already satisfied and the maximum is still 18. A minimum only reduces the maximum flow when it forces flow along a route the optimum would otherwise avoid. [4]