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ANSWER BOOK

Mean, variance and skewness of continuous variables

Further Statistics 2 · Year FM

6 questions · 21 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $E(X) = \int x^2/8 \, dx = [x^3/24]$ from 0 to 4 = 64/24 = 8/3, about 2.667. $E(X^2) = \int x^3/8 \, dx = [x^4/32] = 256/32 = 8$. So $\mathrm{Var}(X) = 8 - (8/3)^2 = 8 - 64/9$ = 8/9, about 0.889. [3]

A2 $E(3X + 2) = 3E(X) + 2 = 3 \times 8/3 + 2$ = 10. $\mathrm{Var}(3X + 2) = 3^2\mathrm{Var}(X) = 9 \times 8/9$ = 8. The shift changes the mean but not the spread, and the scale factor squares in the variance. [3]

SECTION B · Standard

B1 $F(x) = x^2/16$, so the median solves $m^2 = 8$ and $m = 2\sqrt{2}$ = 2.828. The density increases throughout the range, so the mode is at the right-hand end, 4. Since mean 2.667 < median 2.828 < mode 4, the distribution is negatively skewed, with its tail running to the left. [4]

B2 $E(X) = 3\int(x - 2x^2 + x^3) \, dx = 3(1/2 - 2/3 + 1/4) = 3/12$ = 0.25. $E(X^2) = 3(1/3 - 1/2 + 1/5) = 3/30 = 0.1$, so $\mathrm{Var}(X) = 0.1 - 0.0625$ = 0.0375. $F(x) = 1 - (1 - x)^3$, so the median solves $(1 - m)^3 = 0.5$, giving m = 0.206. The mode is 0, so mode < median < mean: positive skew. [4]

B3 $E(1/X) = \int (1/x)(x/8) \, dx$ from 0 to 4 = $\int 1/8 \, dx = [x/8]$ = 0.5. Note that this is not $1/E(X)$, which would be 0.375: the expectation of a function is not the function of the expectation. [3]

SECTION C · Stretch

C1 $E(aX + b) = \int(ax + b)f(x) \, dx = a\int xf(x) \, dx + b\int f(x) \, dx = aE(X) + b$, using the fact that the density integrates to 1. Then $\mathrm{Var}(aX + b) = E([aX + b - aE(X) - b]^2) = E(a^2[X - E(X)]^2) = a^2E([X - E(X)]^2) = a^2\mathrm{Var}(X)$. The constant b disappears because it shifts the variable and its mean equally, leaving the deviations untouched. [4]