



ANSWER BOOK

Minimum spanning trees: Prim and Kruskal

Decision Mathematics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

- A1** A subset of the edges that connects every node with no cycle, chosen to have the least possible total weight. It always has exactly $n - 1$ edges: any fewer leaves the network disconnected and any more creates a cycle. [3]
- A2** **Prim**: start at any node and repeatedly add the cheapest edge joining the growing tree to a node not yet in it. **Kruskal**: sort the edges by weight and take them in order, rejecting any that would close a cycle. Both always produce a minimum spanning tree, so the totals always agree. [3]

SECTION B · Standard

- B1** From A: AB (5). From {A, B}: the choices are AC 7, BC 4, AD 9, BE 9, so take BC (4). From {A, B, C}: CD (3). Then CE (6) beats BE 9. Finally EF (2) beats DF 10. Order: **AB, BC, CD, CE, EF**, total **20**, five edges for six nodes. [4]
- B2** Sorted: EF 2, CD 3, BC 4, AB 5, CE 6, AC 7, BE 9, AD 9, DF 10. Take EF, CD, BC, AB, CE. Then **AC is rejected** (A, B and C are already joined), **BE rejected** (B and E already joined through C), **AD rejected** and **DF rejected**, each for the same reason. Total **20**: the same tree Prim found. [4]
- B3** Prim only ever looks at the columns of the nodes already in the tree, which is exactly what a matrix makes easy: cross out the row of each node as it joins and scan the joined columns for the smallest uncrossed entry. Kruskal would require sorting every entry of the matrix first and then checking each edge for cycles, which is far more work by hand. [3]

SECTION C · Stretch

- C1** Both algorithms will still produce a minimum spanning tree of **7** edges, and the totals will agree however the tie is broken, since any minimum spanning tree has the same weight. But the trees themselves may differ: the two algorithms may reach the tied edges at different points and take different ones. Ties are the only circumstance in which the two can produce different trees. [4]