inkMaths

ANSWER BOOK

Mixed strategies

Decision Mathematics 2 · Year FM

6 questions · 24 marks

NAVY EDITION

SECTION A · Warm-up

A1 If maximin and minimax differ, then whichever single pair of choices is made, one player can improve by switching. A predictable choice can be exploited, so the row player has to vary between rows with fixed probabilities. Being unpredictable in the right proportion is what raises the guaranteed pay-off above the maximin. [2]

A2 Equate the two: 2 + 5p = 7 - 3p, so 8p = 5 and p = 0.625. The value is 2 + 5(0.625) = 5.125, and 7 - 3(0.625) = 5.125 confirms it. [3]

SECTION B · Standard

B1 Row minima are 2 and 3, so the maximin is 3. Column maxima are 6 and 7, so the minimax is 6. They differ, so there is no stable solution.
Let the row player play R1 with probability p. Against C1 the expected pay-off is 6p + 3(1 - p) = 3 + 3p; against C2 it is 2p + 7(1 - p) = 7 - 5p.
Equating: 3 + 3p = 7 - 5p, so 8p = 4 and p = 0.5.
The value is 3 + 3(0.5) = 4.5, and 7 - 5(0.5) = 4.5 agrees. The row player plays each row half the time, and 4.5 lies between the maximin of 3 and the minimax of 6 as it must. [6]

B2 Let the column player play C1 with probability q. Against R1 the expected loss is 6q + 2(1 - q) = 2 + 4q; against R2 it is 3q + 7(1 - q) = 7 - 4q.
Equating: 2 + 4q = 7 - 4q, so 8q = 5 and q = 0.625.
The value is 2 + 4(0.625) = 4.5, matching the row player's calculation, so both mixes are right. Note that the two probabilities differ even though the value does not. [4]

B3 The proportions are fixed by where the two expected pay-off lines cross, not by the sizes of the entries. Here the answer is an even split, despite R2 having the larger total. Favouring R2 would let the column player respond with C1, where R2 pays only 3, dragging the guaranteed pay-off below 4.5. [3]

SECTION C · Stretch

C1 With R1 played with probability p, the expected pay-offs are $2 + 2p$ against C1, $7 - 6p$ against C2, and $3 + 3p$ against C3. Sketch all three over 0 to 1 and take the highest point of the lower boundary.
At $p = 0$ the values are 2, 7 and 3; at $p = 1$ they are 4, 1 and 6. The lower boundary is C1 rising and C2 falling, and they meet where $2 + 2p = 7 - 6p$, so $8p = 5$ and $p =$ 0.625, giving a value of 3.25.
There C3 is worth $3 + 3(0.625) = 4.875$, above the boundary, so C3 is never used: it would hand the row player more than 3.25.
A 3 by 3 game has no single p to plot, so it is converted into a linear program: add a constant to every entry to make them positive if needed, maximise the value subject to one constraint per opposing choice, solve by Simplex, then subtract the constant again. [6]