inkMaths

ANSWER BOOK

Modelling with differential equations

Differential equations · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The y term is the restoring force pulling back towards equilibrium, its coefficient the stiffness; the y' term is resistance, draining energy in proportion to speed. [2]

A2 Discriminant 64 - 100 = -36 < 0: light damping. The roots are -4 ± 3i, so the motion oscillates at angular frequency 3 inside a decaying envelope e^{-4t}: frequency from the imaginary part, decay from the real part. [3]

SECTION B · Standard

B1 Discriminant 100 - 100 = 0: critical damping, repeated root m = -5, solution $y = (A + Bt)e^{-5t}$. Critical damping returns the system to rest as fast as possible with no overshoot; lighter damping bounces, heavier creeps. [4]

B2 Discriminant 25 - 16 = 9 > 0: heavy damping, real roots -1 and -4. The solution $Ae^{-t} + Be^{-4t}$ creeps back to equilibrium without ever oscillating, the slower exponential setting the pace at late times. [3]

B3 Differentiate the first equation: x'' = 2 dy/dt = -4x, so x'' + 4x = 0 and x = A cos 2t + B sin 2t. x(0) = 3 gives A = 3; y = x'/2 = -A sin 2t + B cos 2t at t = 0 gives B = 0. So x = 3 cos 2t and y = -3 sin 2t: the pair orbit a circle of radius 3 at angular speed 2. [5]

SECTION C · Stretch

C1 At b = 0 the motion is simple harmonic, oscillating for ever. Small b gives light damping: the same oscillation inside a shrinking envelope. At $b^2 = 4c$ the oscillation disappears exactly, the critical case and quickest settle. Beyond that, heavy damping: two real decay rates and a slow creep home. [3]