inkMaths

ANSWER BOOK

Modulus, argument and loci

Complex numbers · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $|z| = \sqrt{(1 + 3)} = 2$. The point $(-1, \sqrt{3})$ sits in the second quadrant, so $\arg z = \pi - \pi/3 = 2\pi/3$. A sketch first; the calculator's arctan alone would report the wrong quadrant. [2]

A2 $x = 2\cos(\pi/3) = 1$ and $y = 2\sin(\pi/3) = \sqrt{3}$, so $z = 1 + \sqrt{3}\,i$. Modulus-argument to cartesian is two right-triangle reads. [3]

SECTION B · Standard

B1 Multiplying multiplies moduli and adds arguments: $|zw| = 6$, $\arg zw = \pi/6 + \pi/4 = 5\pi/12$. Dividing divides and subtracts: $|z/w| = 2/3$, $\arg(z/w) = \pi/6 - \pi/4 = -\pi/12$. No cartesian expansion is needed anywhere. [4]

B2 A circle of radius 2 centred at $3 + 4i$, the point $(3, 4)$. The centre is distance 5 from the origin, so $|z|$ runs from $5 - 2 = 3$ to $5 + 2 = 7$, along the line joining the origin to the centre. Centre distance plus and minus radius, once the sketch is drawn. [4]

B3 Equal distance from 2 and from -2: the perpendicular bisector of the segment joining $(2, 0)$ and $(-2, 0)$, which is the imaginary axis. Any equation $|z - a| = |z - b|$ is the perpendicular bisector of a and b. [3]

SECTION C · Stretch

C1 $|z| \leq 3$ is the disc of radius 3; the argument condition keeps the first quadrant, including both axes' edges. The region is a quarter disc, area $(1/4)\pi(3^2) = 9\pi/4$. Loci intersect exactly as their sketches suggest: draw both, then read the overlap. [4]