inkMaths

ANSWER BOOK

Motion in a vertical circle

Further Mechanics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The tension always acts along the string, which is perpendicular to the direction of motion at every instant, so it does no work. Only gravity does work, and it is conservative, so the total of kinetic and potential energy is constant. [2]

A2 At the critical speed the string is just taut, so $T = 0$ and the only radial force is the weight: $mg = mv^2/r$. Hence $v^2 = gr = 9.8 \times 1.2 =$ 11.76, so $v =$ 3.43 m/s. [3]

SECTION B · Standard

B1 Energy from the bottom to the top, a rise of $2r$: $u^2 = v^2 + 4gr$. With $v^2 = gr$ at the critical case, $u^2 = 5gr = 5 \times 9.8 \times 1.2 =$ 58.8, so $u =$ 7.67 m/s. The $u^2 = 5gr$ result is worth remembering, but the derivation carries the marks. [4]

B2 It falls a height equal to the radius, so $v^2 = 2gr = 2(9.8)(2) = 39.2$ and $v =$ 6.26 m/s. At the lowest point the radial equation is $R - mg = mv^2/r$, so per unit mass $R = 9.8 + 39.2/2 =$ 29.4 N: three times the weight. [4]

B3 A rod can push outwards as well as pull inwards, so the constraint never fails and the particle only has to reach the top with some speed: v > 0, giving u^2 > $4gr$. A string can only pull, so it needs the tension to remain non-negative all the way round, which forces $v^2 \geq gr$ at the top and $u^2 \geq 5gr$ at the bottom. [3]

SECTION C · Stretch

C1 At angle θ the radial equation is $mg\cos\theta - R = mv^2/r$. Energy from the top, having dropped $r(1 - \cos\theta)$: $v^2 = 2gr(1 - \cos\theta)$. The particle leaves when $R = 0$, so $g\cos\theta = v^2/r = 2g(1 - \cos\theta)$. Hence $3\cos\theta = 2$ and cos θ = 2/3, that is $\theta = 48.2°$. The result is independent of r and of the mass, so every smooth sphere sheds a particle at the same place. [4]