inkMaths

ANSWER BOOK

Newton-Raphson and the trapezium rule

Numerical methods · Year 13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $x_{n+1} = x_n - f(x_n)/f'(x_n)$: slide down the tangent at the current point to where it crosses the axis, and start again from there. [2]

A2 Area ≈ $(h/2)[y_0 + y_n + 2(y_1 + \ldots + y_{n-1})]$: the ends count once, everything between counts twice, because each interior ordinate belongs to two trapezia. [2]

SECTION B · Standard

B1 $f(1.5) = -0.125$ and $f'(1.5) = 3(1.5)^2 + 1 = 7.75$, so $x_1 = 1.5 + 0.125/7.75 = 1.516129\ldots$ Then $f(x_1) = 0.00115\ldots$, $f'(x_1) = 7.8961\ldots$, and $x_2 = 1.51598$ (5 d.p.). Two steps already agree to four decimal places: Newton-Raphson roughly doubles the correct digits each time. [4]

B2 $h = 0.25$ and the ordinates are $\ln 1 = 0$, $\ln 1.25 = 0.2231$, $\ln 1.5 = 0.4055$, $\ln 1.75 = 0.5596$, $\ln 2 = 0.6931$. Area ≈ $(0.25/2)[0 + 0.6931 + 2(0.2231 + 0.4055 + 0.5596)] = 0.125 \times 3.0696 = 0.3837$. [4]

B3 $\ln x$ is concave: it bends over its chords, so every trapezium's sloped top runs below the curve and each strip undershoots. For a convex curve the same reasoning flips and the rule overestimates. [2]

SECTION C · Stretch

C1 The formula divides by $f'(x_n)$. Near a stationary point f' is close to zero, so the correction f/f' is enormous: the tangent is nearly horizontal and meets the axis far away, throwing the next iterate out of the region of interest, or the method breaks down entirely when $f' = 0$ exactly. [3]

C2 Double the number of strips: more, narrower trapezia hug the curve more closely. For $\ln x$ the estimates should climb towards 0.3863 from below as strips are added, each undershoot smaller than the last, because the curve stays concave across the whole interval. [3]