



QUESTION WORKBOOK

Newton–Raphson and the trapezium rule

Numerical methods · Year 13

7 questions · 20 marks

NAVY SCREEN EDITION

SECTION A · Warm-up

A1 Write down the Newton-Raphson iteration formula for solving $f(x) = 0$. [2]

A2 The trapezium rule with n strips of width h estimates an integral from the ordinates $y_0, y_1, \dots, y_n$. Write down the rule. [2]

SECTION B · Standard

B1 $f(x) = x^3 + x - 5$. Using Newton-Raphson with $x_0 = 1.5$, find x_1 and x_2 , giving x_2 to 5 decimal places. [4]

B2 Use the trapezium rule with 4 strips to estimate the integral of $\ln x$ between $x = 1$ and $x = 2$, to 4 decimal places. [4]

- B3** The exact value of that integral is $2 \ln 2 - 1 = 0.3863$. Explain, with a sketch in mind, why the trapezium estimate came out too small. [2]

SECTION C · Stretch

- C1** Newton–Raphson fails badly if the starting value is chosen near a stationary point of f . Explain why, using the formula. [3]

- C2** State one way to improve a trapezium–rule estimate, and one check that the improved estimate behaves as expected for the $\ln x$ integral above. [3]
