inkMaths

ANSWER BOOK

Newton's laws with a variable force

Further Mechanics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 dv/dt and $v\,dv/dx$, which are equal by the chain rule. Use dv/dt when the force depends on time or on velocity and time is wanted; use $v\,dv/dx$ when it depends on position, or on velocity and distance is wanted. The aim is an equation that separates into two variables. [2]

A2 The force depends on x, so use $3v\,dv/dx = 12 - 3x$, that is $v\,dv/dx = 4 - x$. Integrating: $\frac{1}{2}v^2 = 4x - \frac{1}{2}x^2 + c$, and $v = 0$ at $x = 0$ gives $c = 0$. So $v^2 = 8x - x^2$, and at $x = 2$, $v^2 = 16 - 4 = 12$, giving $v =$ 3.46 m/s. [3]

SECTION B · Standard

B1 Again the force depends on x: $2v\,dv/dx = 6/(x + 1)^2$. Integrating: $v^2 = \int 6/(x + 1)^2\,dx = -6/(x + 1) + c$. With $v = 0$ at $x = 0$, $c = 6$, so $v^2 = 6 - 6/(x + 1)$. At $x = 2$: $v^2 = 6 - 2 = 4$ and $v =$ 2 m/s. However far it travels the speed cannot exceed $\sqrt{6}$, since the force dies away too fast. [4]

B2 At the surface the force per unit mass is g, so $GM = gR^2$ and at distance x it is gR^2/x^2. Work $= \int gR^2/x^2\,dx$ from R to $3R$ $= gR^2[-1/x] = gR^2(1/R - 1/3R) =$ 2gR/3. With Earth values that is about 6.53 million joules per kilogram, two thirds of the energy needed to escape completely. [4]

B3 Those equations are derived by integrating a constant acceleration, so they are simply not solutions of the problem when a varies. Using the average force gives the right total impulse only if the force is averaged over time, and the right work only if it is averaged over distance, and those are different averages. Neither gives the correct answer for both. [3]

SECTION C · Stretch

C1 Using $v\,dv/dx = -gR^2/x^2$ and integrating from the surface: $\frac{1}{2}v^2 = gR^2/x + c$. The escape condition is that v tends to 0 as x grows without bound, so $c = 0$ and at the surface $\frac{1}{2}v^2 = gR$. Hence $v =$ √(2gR). Substituting: $\sqrt{2 \times 9.8 \times 6.4 \times 10^6} =$ 11200 m/s, about 11.2 km/s. The mass of the escaping body plays no part. [4]