inkMaths

ANSWER BOOK

Numerical methods for differential equations

Further Pure 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Forward: $(y_{n+1} - y_n)/h$. Central: $(y_{n+1} - y_{n-1})/2h$. The central difference is the more accurate for the same step length, because it straddles the point rather than looking only ahead. [2]

A2 First step: $y_1 = 1 + 0.1(0 + 1) = 1.1$. Second step: $y_2 = 1.1 + 0.1(0.01 + 1.1) = 1.211$. Each step reuses the previous value, so an early rounding error propagates through everything after it. [3]

SECTION B · Standard

B1 The approximation is $(y_{n+1} - 2y_n + y_{n-1})/h^2$. Rearranged: $y_{n+1} = 2y_n - y_{n-1} + h^2$ times the second derivative at x_n. Two previous values are needed, which is why second order problems always supply two starting conditions. [4]

B2 $h = 0.25$ and the ordinates are 1, 0.939413, 0.778801, 0.569783, 0.367879. The weighted sum with pattern 1, 4, 2, 4, 1 is 8.962265, and multiplying by $h/3$ gives 0.7469. The true value is 0.7468, so four strips are accurate to three decimal places on a function with no elementary antiderivative. [4]

B3 The rule fits a parabola through each consecutive group of three ordinates, which uses up two strips at a time. An odd number leaves one strip with no partner and no parabola to fit, so the rule cannot be applied as it stands. [3]

SECTION C · Stretch

C1 Rearranged: $y_{n+1} = 2y_n - y_{n-1} + h^2(2x_n - y_n)$. At $n = 1$: $y_2 = 2.1 - 1 + 0.01(0.2 - 1.05) = 1.09150$. At $n = 2$: $y_3 = 2(1.0915) - 1.05 + 0.01(0.4 - 1.0915) = 1.12609$. The recurrence walks forward two values at a time, each step needing only the two before it. [4]