inkMaths

ANSWER BOOK

Oscillations on strings and springs

Further Mechanics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 At equilibrium the tension already balances the weight. Measuring the displacement from there, the extra tension is $\lambda x/l$ and the weight has cancelled out, leaving a restoring force proportional to x. The weight therefore fixes where the oscillation is centred but has no effect on the motion about that point. [2]

A2 $\omega^2 = \lambda/(ml) = 40/(1 \times 0.5) =$ 80, so $\omega =$ 8.94 rad/s and $T = 2\pi/\omega =$ 0.702 s. [3]

SECTION B · Standard

B1 At equilibrium the tension equals the weight: $40e/0.5 = 1(9.8)$, so $80e = 9.8$ and $e =$ 0.1225 m. Note that $\omega^2 = \lambda/(ml) = g/e$, which is a useful check: $9.8/0.1225 = 80$. [4]

B2 Released from rest at 0.1225 m above equilibrium, so the amplitude is 0.1225 m. The greatest speed is $a\omega = 0.1225 \times 8.944 =$ 1.10 m/s, reached at the equilibrium position, where the net force is zero and the acceleration vanishes. [4]

B3 A string cannot push, so above the natural length it goes slack and exerts nothing. Released from exactly the natural length, the motion is still simple harmonic throughout, since the particle only just reaches that point. Any greater amplitude and the particle would rise above it as a free body under gravity, so the motion would be simple harmonic for part of each cycle only. [3]

SECTION C · Stretch

C1 Displacing the particle by x stretches one spring by an extra x and shortens the other by x. Both then push it back, each with force $25x/0.4$, so the total restoring force is $2 \times 25x/0.4 = 125x$. Hence $0.5\ddot{x} = -125x$, giving $\omega^2 = 2\lambda/(ml) =$ 250 and $\omega = 15.8$ rad/s. $T = 2\pi/15.8 =$ 0.397 s: two springs make the system stiffer and the period shorter by a factor of $\sqrt{2}$. [4]