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ANSWER BOOK

Parametric equations

Coordinate geometry · Year 12–13

7 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $x = 4 + 1 = 5$ and $y = 4 - 1 = 3$: the point $(5, 3)$. One value of the parameter, one point of the curve. [2]

A2 The simpler equation gives $t = x + 3$; substituting into the other, $y = (x + 3)^2$. Solve the easier equation for t and feed the harder one: that route works for almost every algebraic pair. [2]

SECTION B · Standard

B1 $t = x/2$, so $y = 12 \div (x/2) = 24/x$, or $xy = 24$: a hyperbola with the axes as asymptotes. The excluded value $t = 0$ matches the curve never reaching $x = 0$, so nothing extra needs removing. [3]

B2 Squaring and adding: $x^2 + y^2 = 25(\cos^2 t + \sin^2 t) = 25$, the circle of radius 5 about the origin. On $0 \leq t \leq \pi$ the value $y = 5 \sin t$ is never negative, so C is the upper semicircle only, running from $(5, 0)$ at $t = 0$ to $(-5, 0)$ at $t = \pi$. Trig pairs convert by identity, and the parameter's range then says how much of the curve is really there. [4]

B3 $\cos t = x/4$ and $\sin t = y/3$, so the identity $\sin^2 t + \cos^2 t = 1$ gives $x^2/16 + y^2/9 = 1$: an ellipse, 4 wide and 3 tall from the centre. Dividing by the coefficients before squaring is the move; squaring first buries them. [3]

SECTION C · Stretch

C1 $y = (t^2)^3$, so $y = x^3$. But $x = t^2 \geq 0$ for every real t, so C is only the half of the cubic with $x \geq 0$. The Cartesian equation says where a point may be; the parametrisation says where the point actually goes. [3]

C2 Landing means $y = 0$: $5t(3 - t) = 0$, so $t = 0$ (launch) or $t = 3$. It lands after 3 seconds, at $x = 20 \times 3 = 60$ metres. One clock drives both coordinates: solve the vertical story for time, then hand the time to the horizontal one. [3]