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ANSWER BOOK

Partial fractions

Algebra and functions · Year 12–13

6 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 [2]

Set it equal to $A/(x + 1) + B/(x + 2)$ and multiply up: $3x + 5 = A(x + 2) + B(x + 1)$. Substituting $x = -1$ kills the B bracket: $2 = A$. Substituting $x = -2$: $-1 = -B$, so $B = 1$. The answer is $2/(x + 1) + 1/(x + 2)$.

A2 [2]

At $x = -1$ the factor $(x + 1)$ is zero, so the whole B term vanishes and the equation collapses to a statement about A alone. Each root of the denominator switches off every constant except its own: that is all there is to it.

SECTION B · Standard

B1 [4]

Set $A/x + B/(x - 1) + C/(x + 2)$ and multiply up: $6x^2 + 5x - 2 = A(x - 1)(x + 2) + Bx(x + 2) + Cx(x - 1)$. Then $x = 0$ gives $-2 = -2A$, so $A = 1$; $x = 1$ gives $9 = 3B$, so $B = 3$; $x = -2$ gives $12 = 6C$, so $C = 2$. The answer is $1/x + 3/(x - 1) + 2/(x + 2)$, and any spare x value, say $x = 2$, checks it: $32 = 4 + 24 + 4$.

B2 [4]

A squared bracket needs both powers: $A/(x + 3) + B/(x + 3)^2$. Multiplying up: $2x + 7 = A(x + 3) + B$. Substituting $x = -3$: $1 = B$. Comparing x coefficients: $A = 2$. So the answer is $2/(x + 3) + 1/(x + 3)^2$. One fraction per bracket fails here; the repeated factor demands the pair.

B3 [4]

The fraction is improper: the top's degree matches the bottom's. Divide first: $x^2 + 4x + 5 = (x^2 + 3x + 2) + (x + 3)$, so the fraction is $1 + (x + 3)/((x + 1)(x + 2))$. Splitting the proper part: $x = -1$ gives $A = 2$, $x = -2$ gives $B = -1$. The answer is $1 + 2/(x + 1) - 1/(x + 2)$. Skipping the division and writing two fractions for an improper top is the standard wrong turn.

SECTION C · Stretch

C1 [4]

Three constants: $A/(x + 1) + B/(x + 1)^2 + C/(x + 2)$. Multiplying up: $3x^2 + 7x + 8 = A(x + 1)(x + 2) + B(x + 2) + C(x + 1)^2$. Then $x = -1$ gives $4 = B$; $x = -2$ gives $6 = C$; comparing x^2 coefficients, $A + C = 3$, so $A = -3$. The answer is $-3/(x + 1) + 4/(x + 1)^2 + 6/(x + 2)$. A negative constant is not a mistake; the check at $x = 0$ confirms it: $8 = -6 + 8 + 6$.