inkMaths

ANSWER BOOK

Polar curves

Polar coordinates · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $x = 4\cos(2\pi/3) = -2$ and $y = 4\sin(2\pi/3) = 2\sqrt{3}$: the point $(-2, 2\sqrt{3})$, in the second quadrant as an argument between $\pi/2$ and π requires. [2]

A2 $r = \sqrt{2}$. The point sits in the fourth quadrant, so $\theta = -\pi/4$. Quoting $+\pi/4$ from arctan without checking the quadrant is the standard slip; the sketch settles it. [3]

SECTION B · Standard

B1 Multiply by r: $r^2 = 6r\sin\theta$, so $x^2 + y^2 = 6y$. Completing the square: $x^2 + (y - 3)^2 = 9$, a circle of radius 3 centred at $(0, 3)$, sitting on the pole. $r = a\sin\theta$ and $r = a\cos\theta$ are always circles through the origin. [4]

B2 $r\sec\theta$ means $r\cos\theta = \ldots$ rearranged: $r = 2/\cos\theta$ gives $r\cos\theta = 2$, which is $x = 2$: a vertical line two units right of the pole. A polar equation this short can still be a perfectly straight line. [3]

B3 $r = 6$, 3 and 0 respectively. The curve bulges to $(6, 0)$ on the initial line, passes three units above the pole, and pinches onto the pole itself at the back: a heart on its side, traced once as θ makes a full turn. [4]

SECTION C · Stretch

C1 Three petals, each of maximum radius 2. The curve reaches the pole when $\cos 3\theta = 0$, first at $3\theta = \pi/2$, so $\theta = \pi/6$. An odd multiple of θ gives that many petals; an even multiple gives twice as many, since the negative-r halves land in new positions. [4]