inkMaths

ANSWER BOOK

Polynomials and the factor theorem

Algebra and functions · Year 12–13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $f(2) = 8 - 16 + 2 + 6 = 0$, so by the factor theorem $(x - 2)$ is a factor. Mind the sign: the factor $(x - 2)$ pairs with the root $x = 2$, not $x = -2$. [2]

A2 Divide leading terms in turn: $x^3 \div x = x^2$, multiply back and subtract, leaving $-2x^2 + x$; then $-2x^2 \div x = -2x$, leaving $-3x + 6$; then -3 exactly. The quotient is $x^2 - 2x - 3$ with remainder zero, as the factor theorem promised. [3]

SECTION B · Standard

B1 The quotient factorises: $x^2 - 2x - 3 = (x - 3)(x + 1)$, so $f(x) = (x - 2)(x - 3)(x + 1)$, with roots 2, 3 and -1. Completely means down to linear brackets: stopping at the quadratic loses the final mark. [3]

B2 Write the dividend with every power present: $2x^3 + 3x^2 + 0x - 5$. Dividing: quotient $2x^2 - x + 2$, remainder -9. The check by substitution agrees: $f(-2) = -16 + 12 - 5 = -9$, which is the remainder theorem doing division without dividing. [4]

B3 The root that kills $(2x - 1)$ is $x = \frac{1}{2}$: $f(\frac{1}{2}) = \frac{1}{4} + 5/4 - \frac{1}{2} - 1 = 0$, so the bracket is a factor. Dividing: $f(x) = (2x - 1)(x^2 + 3x + 1)$. For a factor $(ax - b)$ the test value is b/a, not b: substituting $x = 1$ here would prove nothing. [4]

SECTION C · Stretch

C1 Factorise top and bottom: $(x - 3)(x + 3)/[(x + 3)(x - 2)] = (x - 3)/(x - 2)$, cancelling the common bracket. The cancellation assumes $x \neq -3$, where the original fraction is undefined; and $x = 2$ stays excluded in both versions. Only whole brackets cancel: crossing out lone x-terms is the error this topic exists to retire. [4]