



QUESTION WORKBOOK

Probability generating functions

Further Statistics 1 · Year FM

6 questions · 20 marks

NAVY SCREEN EDITION

SECTION A · Warm-up

A1 Define the probability generating function of a discrete random variable X , and state the value of $G(1)$. [2]

A2 X has $G(t) = (0.6 + 0.4t)^8$. Name the distribution and find $E(X)$. [3]

SECTION B · Standard

B1 For the same X , find $\text{Var}(X)$ using the generating function. [4]

- B2** Derive the probability generating function of $X \sim \text{Po}(\lambda)$, and use it to confirm $E(X) = \lambda$. [4]

- B3** X and Y are independent with $X \sim \text{Po}(2)$ and $Y \sim \text{Po}(7)$. Use generating functions to identify the distribution of $X + Y$. [3]

SECTION C · Stretch

- C1** A geometric distribution has $G(t) = pt/(1 - qt)$ with $q = 1 - p$. Show that the variance is q/p^2 . [4]