InkMaths

ANSWER BOOK

Quadratic functions

Algebra and functions · Year 12–13

7 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Half the 8, square it, correct: $(x + 4)^2 - 16 + 3 = (x + 4)^2 - 13$. The square is smallest when its bracket is zero, so the minimum point is $(-4, -13)$. [2]

A2 $b^2 - 4ac = 9 - 40 = -31$, which is negative: no real roots. The parabola opens upward and its minimum sits above the axis, so it never touches down. [2]

SECTION B · Standard

B1 Take the 3 out of the x terms first: $3(x^2 - 4x) + 5 = 3[(x - 2)^2 - 4] + 5$. Distributing: $3(x - 2)^2 - 12 + 5 = 3(x - 2)^2 - 7$. Minimum point $(2, -7)$. The correction term gets multiplied by the 3 on the way back out, which is where the marks are lost. [4]

B2 Repeated root means discriminant zero: $k^2 - 100 = 0$, so $k = 10$ or -10. Two different parabolas, each just touching the axis, one with its touch to the left, one to the right. [3]

B3 Two distinct roots need $16 - 4k$ > 0, so k < 4. But $k = 0$ must also be excluded: the equation would then be linear, with one root, not two. So k < 4 and $k \neq 0$. The vanishing leading coefficient is the part most answers miss. [3]

B4 Since $9^x = (3^x)^2$, substitute $u = 3^x$: $u^2 - 10u + 9 = 0$, factorising as $(u - 1)(u - 9) = 0$. Then $3^x = 1$ gives $x = 0$, and $3^x = 9$ gives $x = 2$. A quadratic in a function of x: spot the disguise, substitute, return. [3]

SECTION C · Stretch

C1 Substitute $u = x^2$: $u^2 - 5u + 4 = 0$, so $(u - 1)(u - 4) = 0$ and $u = 1$ or 4. Each positive u value gives two x values: $x = \pm 1$ and $x = \pm 2$, four roots in all. Stopping at $u = 1, 4$ leaves the question half done: the return journey is where the last mark lives. [4]