inkMaths

ANSWER BOOK

Radians, arcs and small angles

Trigonometry · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $60° = \pi/3$, since $180° = \pi$. And $3\pi/4 = 3 \times 45° = 135°$. Multiples of π over small denominators cover every angle the exact-value table knows. [2]

A2 Arc: $s = r\theta = 5 \times 0.8 = 4$ cm. Area: $\frac{1}{2}r^2\theta = \frac{1}{2} \times 25 \times 0.8 = 10$ cm^2. No factor of 360 in sight, which is the whole reward for working in radians. [2]

SECTION B · Standard

B1 The arc is $9 \times 1.4 = 12.6$ cm, and the perimeter adds the two radii: $12.6 + 18 = 30.6$ cm. The area is $\frac{1}{2} \times 81 \times 1.4 = 56.7$ cm^2. Forgetting the two straight edges in the perimeter is the standard slip. [3]

B2 From $s = r\theta$: $\theta = 12/7.5 = 1.6$ radians. Then the area is $\frac{1}{2} \times 7.5^2 \times 1.6 = 45$ cm^2. The formulae run backwards as happily as forwards; the radian answer needs no unit conversion before entering the area formula. [3]

B3 Segment = sector - triangle: $\frac{1}{2} \times 36 \times 1.2 - \frac{1}{2} \times 36 \times \sin 1.2 = 21.6 - 16.78 = 4.82$ cm^2. The triangle uses the area formula $\frac{1}{2}r^2 \sin\theta$ with the two radii as its sides; the subtraction finishes the job. [3]

SECTION C · Stretch

C1 $\cos 2\theta \approx 1 - (2\theta)^2/2 = 1 - 2\theta^2$, so the top is approximately $2\theta^2$. The bottom is $\theta \times \sin\theta \approx \theta \times \theta = \theta^2$. The ratio is $2\theta^2/\theta^2 = 2$, independent of θ. The replacement for cos must use the angle actually present, 2θ, squared whole: $(2\theta)^2 = 4\theta^2$, not $2\theta^2$. [4]

C2 $\sin 0.1 = 0.0998\ldots$, within a fifth of a percent of 0.1. In radians a small angle's arc and sine are nearly the same length, which is what the approximation says. In degrees the claim reads $\sin 0.1° \approx 0.1$, but $\sin 0.1° = 0.0017\ldots$: the number 0.1 no longer measures the arc, so the geometry behind the approximation is gone. [3]