inkMaths

ANSWER BOOK

Rates of change and building differential equations

Differentiation · Year 12–13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $dV/dt = dV/dr \times dr/dt$. Questions usually give the time rate of one quantity (how fast volume is being pumped in, say) and ask for the time rate of the other; the middle factor dV/dr comes from differentiating the linking formula. [2]

A2 $A = \pi r^2$, so $dA/dr = 2\pi r$. The chain gives $10 = 2\pi \times 4 \times dr/dt$, so $dr/dt = 10/(8\pi) = 5/(4\pi) \approx 0.398$ m per hour. The same area arrives each hour, but it spreads around an ever longer rim, so the radius slows. [3]

SECTION B · Standard

B1 $V = (4/3)\pi r^3$, so $dV/dr = 4\pi r^2 = 64\pi$ at $r = 4$. Then $120 = 64\pi \times dr/dt$, so $dr/dt = 15/(8\pi) \approx 0.597$ cm per second. The linking formula is differentiated with respect to r, not t: the chain supplies the time part. [4]

B2 $V = x^3$ gives $dV/dx = 3x^2 = 27$ at $x = 3$, so $dx/dt = 27/27 = 1$ cm per second. For the surface, $S = 6x^2$ gives $dS/dx = 12x = 36$, so $dS/dt = 36 \times 1 = 36$ cm^2 per second. One edge rate feeds both answers: every rate in the cube routes through dx/dt. [4]

B3 $dP/dt = kP$ with k > 0. The sentence names the rate, names what it is proportional to, and the constant carries the proportionality; stating its sign is part of the model. [3]

SECTION C · Stretch

C1 The rate of decrease of T is $-dT/dt$, and the excess is $T - 20$, so $-dT/dt = k(T - 20)$, that is $dT/dt = -k(T - 20)$, with k > 0. The minus sign and the positive k together keep the drink cooling while it is hotter than the room, and the equation correctly stalls at $T = 20$: sentences into symbols, one clause at a time. [4]