inkMaths

ANSWER BOOK

Reciprocal and inverse trigonometric functions

Trigonometry · Year 12-13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 sec x = 1/cos x, cosec x = 1/sin x, cot x = 1/tan x = cos x/sin x. Then sec 60° = 1/(½) = 2 and cot 45° = 1/1 = 1. The letter after the co- pairs each new function with its parent: the s of sec with cos is the classic mismatch to watch. [2]

A2 Divide every term by $\cos^2\theta$: $\sin^2\theta/\cos^2\theta + 1 = 1/\cos^2\theta$, which reads $\tan^2\theta + 1 = \sec^2\theta$. One line, and papers ask for exactly this line; dividing by $\sin^2\theta$ instead gives the cosec twin. [2]

SECTION B · Standard

B1 $\sec^2\theta = 1 + 25/144 = 169/144$, so sec θ = 13/12, taking the positive root because θ is acute. Then cos θ is its reciprocal, 12/13. No triangle drawn, no angle found: the identity moves straight between the ratios. [3]

B2 $\cot^2\theta = \text{cosec}^2\theta - 1 = 25/16 - 1 = 9/16$, so cot θ = 3/4, positive for acute θ. Since sin θ = 4/5 and cos θ = cot θ × sin θ, cos θ = (3/4)(4/5) = 3/5. The 3-4-5 triangle was underneath all along. [3]

B3 sec x = 2 means cos x = ½, so x = 60° or 300°. Every reciprocal equation is solved by flipping back to the parent function first; trying to work with sec directly just obscures the familiar graph. [3]

SECTION C · Stretch

C1 arcsin returns values in [−π/2, π/2]; arccos returns values in [0, π]. Then arcsin ½ = π/6, arccos (−1) = π, and arctan 1 = π/4. The restricted ranges are what make the inverses functions at all: each must pick one angle from infinitely many candidates, and these are the agreed windows. [4]

C2 cot x = √3 means tan x = 1/√3, so x = 30°, and tangent's 180° period adds 210°. Two roots: 30° and 210°. Flipping cot to tan first keeps everything inside the standard exact-value table. [3]