



ANSWER BOOK

Recurrence relations

Further Pure 2 · Year FM

6 questions · 21 marks

NAVY EDITION

SECTION A · Warm-up

- A1** Complementary function: $A4^n$. For the constant, try $u_n = c$: $c = 4c - 3$ gives $c = 1$. So $u_n = \frac{1}{4}A4^n + 1$, and $u_1 = 2$ gives $4A + 1 = 2$, so $A = 1/4$. Hence $u_n = 4^{n-1} + 1$. The first four terms are 2, 5, 17, 65 either way.
- A2** The right-hand side is linear in n , so try a particular solution $an + b$: $a(n+1) + b = 2(an+b) + n$. Comparing coefficients of n gives $a = 2a + 1$, so $a = -1$; the constants give $a + b = 2b$, so $b = -1$. With the complementary function $A2^n$, $u_1 = 1$ gives $2A - 2 = 1$, so $A = 3/2$ and $u_n = 3(2^{n-1}) - n - 1$. Checking $n = 3$: $12 - 4 = 8$, and the recurrence gives 8 too.

SECTION B · Standard

- B1** Auxiliary equation $m^2 - 5m + 6 = 0$, so $m = 2$ or $m = 3$ and $u_n = A2^n + B3^n$. The conditions give $2A + 3B = 1$ and $4A + 9B = 4$, so $B = 2/3$ and $A = -1/2$. Tidying, $u_n = 2(3^{n-1}) - 2^{n-1}$. Checking $n = 3$: $18 - 4 = 14$, and $5(4) - 6(1) = 14$.
- B2** Auxiliary equation $m^2 - 6m + 9 = 0$ has the repeated root $m = 3$, so the general solution needs the extra factor of n : $u_n = (A + Bn)3^n$. The conditions give $3(A + B) = 3$ and $9(A + 2B) = 18$, so $A + B = 1$ and $A + 2B = 2$, giving $B = 1$ and $A = 0$. Hence $u_n = n3^n$. Checking $n = 3$: 81 , and $6(18) - 9(3) = 81$.
- B3** Base case: $4^0 + 1 = 2$, as given. Assume $u_k = 4^{k-1} + 1$. Then $u_{k+1} = 4(4^{k-1} + 1) - 3 = 4^k + 4 - 3 = 4^k + 1$, which is the formula at $n = k + 1$. True at $n = 1$ and inherited at each step, so true for all positive integers n .

SECTION C · Stretch

- C1** Auxiliary equation $m^2 + m - 6 = 0$ gives $m = 2$ or $m = -3$. For the constant 12, try $u_n = k$: $k + k - 6k = -4k = 12$, so $k = -3$. Then $u_n = A2^n + B(-3)^n - 3$, and the conditions give $2A - 3B = -1$ and $4A + 9B = 13$, so $A = 1$ and $B = 1$. Hence $u_n = 2^n + (-3)^n - 3$. Checking $n = 3$: $8 - 27 - 3 = -22$, and $12 - 10 + 6(-4) = -22$.