inkMaths

ANSWER BOOK

Roots of polynomials

Further algebra and series · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Divide by 2 first: $z^3 + 3z^2 - 2z + 5 = 0$. Then $\alpha + \beta + \gamma = -3$ and $\alpha\beta\gamma = -5$. Skipping the divide is the standard way to lose both marks. [2]

A2 $\alpha + \beta = 6$ and $\alpha\beta = 4$ straight from the coefficients. Then $1/\alpha + 1/\beta = (\alpha + \beta)/(\alpha\beta) = 6/4 = 3/2$: combine over a common denominator and both known quantities appear. [3]

SECTION B · Standard

B1 $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 36 - 8 = 28$, and $\alpha^2\beta^2 = (\alpha\beta)^2 = 16$. A quadratic with those roots is $z^2 - 28z + 16 = 0$: z^2 minus (sum)z plus product, with the symmetric functions doing all the work. [4]

B2 $2^3 - 16 + 2 + 6 = 0$, so $z - 2$ is a factor: the cubic is $(z - 2)(z^2 - 2z - 3) = (z - 2)(z - 3)(z + 1)$, roots 2, 3, −1. Sum: $4 = -(-4)$; product: $-6 = -6/1$. Both relations hold, a worthwhile check on the factorising. [4]

B3 Substitute $w = z + 1$, so $z = w - 1$: $(w - 1)^2 - 6(w - 1) + 4 = w^2 - 8w + 11 = 0$. Alternatively: new sum $6 + 2 = 8$, new product $\alpha\beta + \alpha + \beta + 1 = 4 + 6 + 1 = 11$. Both routes give $z^2 - 8z + 11 = 0$. [4]

SECTION C · Stretch

C1 The coefficients are read off with alternating signs: z^3 − (sum)z^2 + (pairs)z − (product) = $z^3 + 0z^2 - 2z + 8$, that is $z^3 - 2z + 8 = 0$. The relations run both ways: roots to coefficients as directly as coefficients to roots. [3]