inkMaths

ANSWER BOOK

Second order equations

Differential equations · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $am^2 + bm + c = 0$. Substituting the trial solution $y = e^{mx}$ puts a factor of e^{mx} in every term; cancelling it leaves the quadratic. [2]

A2 Auxiliary: $m^2 - 7m + 12 = (m - 3)(m - 4) = 0$, roots 3 and 4. General solution $y = Ae^{3x} + Be^{4x}$: two distinct real roots, two independent exponentials. [3]

SECTION B · Standard

B1 $m^2 + 9 = 0$ gives $m = \pm 3i$, so $y = A \cos 3x + B \sin 3x$. Pure oscillation at angular frequency 3: every solution repeats with period $2\pi/3$. [3]

B2 $m^2 - 6m + 9 = (m - 3)^2 = 0$: a repeated root. The solution needs its extra factor of x: $y = (A + Bx)e^{3x}$. Without the Bx the two constants would collapse into one and two initial conditions could not be met. [3]

B3 $m^2 - 2m + 5 = 0$ gives $m = 1 \pm 2i$, so $y = e^x(A \cos 2x + B \sin 2x)$. $y(0) = 2$ gives $A = 2$. Differentiating and setting $x = 0$: $y'(0) = A + 2B = 0$, so $B = -1$. The solution is $y = e^x(2 \cos 2x - \sin 2x)$: an oscillation at frequency 2 growing inside e^x. [5]

SECTION C · Stretch

C1 $m^2 - 1 = 0$ gives $m = \pm 1$: $y = Ae^x + Be^{-x}$. The conditions give $A + B = 1$ and $A - B = 0$, so $A = B = 1/2$ and $y = (e^x + e^{-x})/2 = \cosh x$. The hyperbolic cosine is what this differential equation looks like when met without its name. [4]