inkMaths

ANSWER BOOK

Sequences and sigma notation

Sequences and series · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The terms are 1, 7, 17. For 71: $2n^2 - 1 = 71$ gives $n^2 = 36$, so $n = 6$, a positive whole number: yes, 71 is the sixth term. Had n come out fractional, the answer would be no, and showing the failed equation would be the proof. [2]

A2 $u_2 = 2 \times 2 + 1 = 5$, then $u_3 = 11$, then $u_4 = 23$. Each feed of the machine should be shown: recurrence questions are marked on the substitutions. [2]

SECTION B · Standard

B1 The terms run 4, 3, 4, 3: the sequence is periodic with period 2, bouncing between two values for ever. Dividing 12 by one value gives the other, so the pair traps itself. Periodic joins increasing and decreasing on the list of behaviours worth naming. [3]

B2 The recipe gives 5, 8, 11, 14, 17, and the Σ says add: the total is 55. Splitting instead as $3(1 + 2 + 3 + 4 + 5) + 2 \times 5 = 45 + 10$ works too, using the fact that adding the constant 2 five times gives 10. [3]

B3 $u_{12} = 50 - 48 = 2$. Negative needs 50 - 4n < 0, so n > 12.5: the first negative term is the 13th, $u_{13} = -2$. The first five terms are 46, 42, 38, 34, 30, which sum to 190. [3]

SECTION C · Stretch

C1 $u_{n+1} - u_n = 2(n + 1)/(n + 2) - 2n/(n + 1) = 2[(n + 1)^2 - n(n + 2)]/[(n + 1)(n + 2)] = 2/[(n + 1)(n + 2)]$, which is positive for every n: increasing. Also u_n = 2 - 2/(n + 1) < 2, since something positive is always subtracted. The sequence climbs for ever yet never reaches 2: increasing does not mean unbounded. [4]

C2 Split the sum: $2(1 + 2 + \ldots + 20) - (1 + 1 + \ldots + 1) = 2 \times 210 - 20 = 400$, using $\Sigma 1 = 20$ for the constant part. And $400 = 20^2$: the first n odd numbers always sum to n^2, each new odd number wrapping another layer round a square. [3]