



ANSWER BOOK

Shortest paths: Dijkstra and Floyd

Decision Mathematics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

- A1** The order in which the node was given a permanent label, its final shortest distance [2] from the start, and the working values tried along the way, each crossed out as a smaller one replaces it. The working values are marked, so they must be shown rather than tidied away.
- A2** Work back from F: $15 - 13 = 2 = \text{weight of EF}$, so EF is on the route. Then $13 - 7 = 6 = \text{CE}$, [3] and $7 - 0 = 7 = \text{AC}$. The route is **A, C, E, F** of length **15**.

SECTION B · Standard

- B1** A, D, F has length $9 + 10 = \text{19}$, against 15 for A, C, E, F. When D was [4] made permanent with label 9, F received the working value 19. Later E was made permanent with 13, and $13 + 2 = 15$ replaced it. The 19 stays on the diagram crossed out, which is the evidence the algorithm was followed.
- B2** It compares the current A to E entry with $7 + 6 = \text{13}$. Since any finite [4] value beats no route, the distance entry becomes **13** and the route entry for A to E is changed to **C**, the first node to head for. With a symmetric matrix the entry from E to A changes in the same way.
- B3** Because all the weights are non-negative, the smallest temporary label anywhere [3] cannot be improved by any later route: reaching it another way would have to pass through a node with an equal or larger label first. Taking the cheapest edge from the current node instead is a greedy walk with no such guarantee, and one short edge into an expensive region can make it go badly wrong.

SECTION C · Stretch

- C1** Floyd runs one iteration per node, and each iteration updates every entry of an n by n [4] matrix, so the work grows like n^3 and every pair is answered at the end. Running Dijkstra once from each of the n nodes also answers every pair and comes to a similar order, but it produces n separate diagrams rather than one table. By hand, Floyd is the more systematic and Dijkstra the quicker if only a few pairs are wanted.