



ANSWER BOOK

Simple harmonic motion

Further Mechanics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

- A1** The acceleration is $-\omega^2 x$, proportional to the displacement from a fixed point and directed back towards it. The speed is greatest at the centre, where $x = 0$ and the acceleration vanishes; the acceleration is greatest at the ends, where the particle is momentarily at rest. [2]
- A2** Maximum speed = $a\omega = 2 \text{ m/s}$. Maximum acceleration = $a\omega^2 = 0.4 \times 25 = 10 \text{ m/s}^2$. Period = $2\pi/\omega = 1.26 \text{ s}$. [3]

SECTION B · Standard

- B1** Using $v^2 = \omega^2(a^2 - x^2)$ twice: $2.56 = \omega^2(a^2 - 0.09)$ and $1.44 = \omega^2(a^2 - 0.16)$. Dividing removes ω^2 : $2.56/1.44 = (a^2 - 0.09)/(a^2 - 0.16)$, so $1.778(a^2 - 0.16) = a^2 - 0.09$ and $0.778a^2 = 0.194$, giving $a^2 = 0.25$ and $a = 0.5 \text{ m}$. Then $\omega^2 = 2.56/0.16 = 16$, so $\omega = 4 \text{ rad/s}$. [4]
- B2** Timing from the centre, $x = 0.5 \sin 4t$. Setting $0.25 = 0.5 \sin 4t$ gives $\sin 4t = 0.5$, so $4t = \pi/6$ and $t = \pi/24 = 0.131 \text{ s}$. The particle returns to the same displacement when $4t = 5\pi/6$, that is $t = 5\pi/24 = 0.654 \text{ s}$, on the way back. [4]
- B3** A larger amplitude means a longer distance to travel, but it also means a proportionally larger restoring force and so a proportionally larger speed at every corresponding point. The two effects cancel exactly, because the force is proportional to displacement, leaving $T = 2\pi/\omega$ whatever the amplitude. It is what makes a pendulum useful as a clock. [3]

SECTION C · Stretch

- C1** Write the acceleration as $v dv/dx$, so $v dv/dx = -\omega^2 x$. Separating: $\int v dv = -\omega^2 \int x dx$, giving $\frac{1}{2}v^2 = -\frac{1}{2}\omega^2 x^2 + c$. At the extreme of the motion, $x = a$ and $v = 0$, so $c = \frac{1}{2}\omega^2 a^2$. Substituting back and doubling: $v^2 = \omega^2(a^2 - x^2)$. The result contains no time, which is exactly why it answers position questions so quickly. [4]