inkMaths

ANSWER BOOK

Solving differential equations

Integration · Year 12–13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Multiply by y and integrate both sides: $\int y\,dy = \int x^2\,dx$. Everything in y moves left with dy, everything in x stays right with dx; the separation is bookkeeping, and the calculus starts after it. [2]

A2 Separate: $\int (1/y)\,dy = \int 3x^2\,dx$, so $\ln y = x^3 + c$. Exponentiating: $y = Ae^{x^3}$ with $A = e^c$. The condition gives A = 2, so $y = 2e^{x^3}$. One constant, absorbed once, pinned once. [3]

SECTION B · Standard

B1 Separate: $\int y^{-2}\,dy = \int dx$, so $-1/y = x + c$. The condition gives c = -1, and rearranging: $y = 1/(1 - x)$. The solution blows up as $x \to 1$ and is valid only for x < 1: a perfectly tame equation growing a vertical asymptote, which is why the validity sentence is part of the answer. [4]

B2 Separate: $\int y\,dy = \int x\,dx$, so $y^2/2 = x^2/2 + c$. The condition gives c = 2, so $y^2 = x^2 + 4$ and $y = \sqrt{(x^2 + 4)}$, the positive root by the given condition. The solution curves are hyperbolas; the condition picks one branch of one of them. [4]

B3 Separating gives $\ln V = -kt + c$, so $V = 500e^{-kt}$ from the initial condition. Then $250 = 500e^{-10k}$ gives $e^{-10k} = ½$, so $k = (\ln 2)/10 = 0.0693$ per minute. The 10-minute halving is this model's half-life, and every further 10 minutes halves the volume again. [4]

SECTION C · Stretch

C1 Separate: $\int h^{-1/2}\,dh = -k\int dt$, so $2\sqrt{h} = -kt + c$. The condition gives c = 8, so $\sqrt{h} = (8 - kt)/2$ and $h = (8 - kt)^2/4$. The depth reaches zero when kt = 8, at t = 8/k: unlike the exponential leak, this tank genuinely empties, in finite time. [3]