



QUESTION WORKBOOK

Subgroups, Lagrange's theorem and isomorphism

Further Pure 2 · Year FM

6 questions · 20 marks

PRINT EDITION

SECTION A · Warm-up

- A1** State Lagrange's theorem and one immediate consequence for the order of an element. [2]

- A2** A cyclic group of order 6 is generated by g . List its subgroups and check them against Lagrange's theorem. [3]

SECTION B · Standard

- B1** A group G has order 8. State the possible orders of its subgroups, and prove that G cannot contain an element of order 3. [4]

- B2** Prove that a group of prime order p is cyclic and has no proper subgroups other than the identity. [4]

- B3** Decide whether the cyclic group of order 4 and the group $\{1, 3, 5, 7\}$ under multiplication modulo 8 are isomorphic, with a reason. [3]

SECTION C · Stretch

- C1** The symmetries of a square form a group of order 8. Find a subgroup of order 4 in two genuinely different ways, and one of order 2. [4]