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ANSWER BOOK

Successive impacts and impacts with a wall

Further Mechanics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The floor is attached to the Earth, so it is an external body and the ball on its own is not an isolated system: the impulse it receives changes its momentum. What still applies is the law of restitution, which says the rebound speed is e times the approach speed. [2]

A2 If the drop height is h, the arrival speed is $\sqrt{(2gh)}$, the rebound speed is $e\sqrt{(2gh)}$, and the height reached is e^2h. So the rebound reaches $0.49 \times 3 =$ 1.47 m. The value of g never has to be worked out. [3]

SECTION B · Standard

B1 The heights form a geometric sequence 3, 1.47, 0.7203, … with ratio 0.49. The first drop is travelled once and every later height twice, so the total is $3 + 2(1.47 + 0.7203 + \ldots) = 3 + 2(1.47/0.51) = 3 + 5.765 =$ 8.76 m. Equivalently $h(1 + e^2)/(1 - e^2)$. [4]

B2 First impact. Momentum: $2(6) = 2v_A + v_B$. Restitution: $v_B - v_A = 3$. Substituting: $3v_A + 3 = 12$, so $v_A =$ 3 m/s and $v_B =$ 6 m/s. At the wall B rebounds at $6/3 =$ 2 m/s back towards A. A is still going forward at 3 and B backward at 2, so they approach at 5 m/s and must meet again. [2]

B3 The rebound speed satisfies $v^2 = 2(9.8)(0.9) = 17.64$, so v = 4.2 m/s. Then $e = 4.2/7 =$ 0.6. [3]

SECTION C · Stretch

C1 The ball arrives at $\sqrt{(2gh)}$ and leaves at $e\sqrt{(2gh)}$, so it rises to e^2h, and repeating gives heights $h, e^2h, e^4h, \ldots$ a geometric sequence with ratio e^2. The first height is travelled once, downwards, and every later height twice, up and down. Total $= h + 2(e^2h + e^4h + \ldots) = h + 2e^2h/(1 - e^2)$, since $|e^2| < 1$. Putting that over a common denominator gives $h(1 - e^2 + 2e^2)/(1 - e^2) = h(1 + e^2)/(1 - e^2)$ as required. As e approaches 1 the total grows without bound, which is what a perfectly elastic ball would do. [4]