inkMaths

ANSWER BOOK

Summing series and the method of differences

Further algebra and series · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $\Sigma r = n(n + 1)/2$ and $\Sigma r^2 = n(n + 1)(2n + 1)/6$, both from $r = 1$ to n. Everything polynomial in [2] this topic reduces to these and the cubes formula.

A2 $\Sigma r^3 = n^2(n + 1)^2/4$, so multiplying by 4 clears the fraction: $\Sigma 4r^3 = n^2(n + 1)^2$. At $n = 3$: $4(1 + 8$ [3] $+ 27) = 144$ and $9 \times 16 = 144$. Agreed.

SECTION B · Standard

B1 Split: $\Sigma r^2 + 2\Sigma r = n(n + 1)(2n + 1)/6 + n(n + 1)$. Take out $n(n + 1)/6$: the bracket is $(2n + 1) +$ [4] $6 = 2n + 7$, giving $n(n + 1)(2n + 7)/6$. Check at $n = 3$: $3 + 8 + 15 = 26$, and $3 \times 4 \times 13/6 = 26$.

B2 The telescope cancels every inner term: the sum is $\frac{1}{2}(1 - 1/(2n + 1)) = n/(2n + 1)$. Check [4] at $n = 3$: $1/3 + 1/15 + 1/35 = 3/7$, and $n/(2n + 1) = 3/7$. Only the first positive and last negative fractions survive.

B3 Subtract two full sums: Σ to $2n$ minus Σ to $n = 2n(2n + 1)(4n + 1)/6 - n(n + 1)(2n + 1)/6 =$ [4] $n(2n + 1)(7n + 1)/6$, after taking out $n(2n + 1)/6$. At $n = 2$: $9 + 16 = 25$ and $2 \times 5 \times 15/6 = 25$. Sums that start above 1 are always a difference of two sums that start at 1.

SECTION C · Stretch

C1 With a gap of two, each term cancels the one two rows below, so two terms survive at [3] each end: $f(1)$, $f(2)$ at the top and $-f(n + 1)$, $-f(n + 2)$ at the bottom. The gap-one telescope leaves only one at each end. In general the number of survivors at each end equals the gap.