inkMaths

ANSWER BOOK

Systems of equations and invariance

Matrices · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 M has rows (2, 1) and (1, 3), v is the column (x, y) and b the column (7, 11). One matrix equation carries both scalar equations at once. [2]

A2 det M = 6 - 1 = 5, so M^{-1} = (1/5) × rows (3, -1), (-1, 2). Then v = M^{-1}b = (1/5)(21 - 11, -7 + 22) = (2, 3). Check: 2 × 2 + 3 = 7 and 2 + 9 = 11. [2]

SECTION B · Standard

B1 Either a prism, where the three pairwise intersection lines are parallel and the planes fence off a triangular tube, or two planes meeting in a line that the third misses everywhere by being parallel to it. The prism is the configuration with three parallel intersection lines; both give an inconsistent system with a singular coefficient matrix. [4]

B2 Take (1, 1) on the line: it maps to (5, 5), still on y = x, and any multiple (t, t) maps to (5t, 5t) by linearity. The line maps into itself with stretch factor 5. Invariant line, not fixed points: every point moves, but none leaves the line. [4]

B3 Try y = mx: the image of (1, m) is (4 + m, 2 + 3m), which lies on y = mx when 2 + 3m = m(4 + m), so $m^2 + m - 2 = 0$, giving m = 1 or m = -2. The new line is y = -2x, stretched by factor 2, since (1, -2) maps to (2, -4). [4]

SECTION C · Stretch

C1 On an invariant line each point maps to a point of the same line, possibly elsewhere; a line of invariant points maps every point to itself. For a reflection, the mirror line is a line of invariant points, while any line perpendicular to the mirror is invariant without its points being fixed. [3]