inkMaths

ANSWER BOOK

Tangents, normals and loci of conics

Further Pure 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The tangent is $ty = x + at^2$, with gradient $1/t$. The normal is perpendicular, so its gradient is $-t$. [2]

A2 Here $4a = 16$ so $a = 4$, and the point is $(at^2, 2at) = (16, 16)$. The tangent $ty = x + at^2$ becomes $2y = x + 16$, that is $y = x/2 + 8$. Checking: $16^2 = 256 = 16 \times 16$, so the point really is on the curve. [3]

SECTION B · Standard

B1 Substituting: $(2x + 2)^2 = 16x$ gives $4x^2 - 8x + 4 = 0$, that is $4(x - 1)^2 = 0$. The repeated root $x = 1$ means the line touches rather than crosses, at the point $(1, 4)$. The condition $c = a/m$ agrees: $a = 4$ and $m = 2$ give $c = 2$. [4]

B2 Differentiating implicitly: $y + x\,dy/dx = 0$, so $dy/dx = -y/x = -1$ at $(4, 4)$. The tangent is $y - 4 = -(x - 4)$, that is $y = 8 - x$. The normal has gradient 1: $y = x$, which is the hyperbola's own axis of symmetry through that point. [4]

B3 A chord $y = 4x + c$ meets the curve where $y^2 = 4(y - c)$, that is $y^2 - 4y + 4c = 0$. The two y-values always sum to 4 regardless of c, so the midpoint has $y = 2$ every time. The locus is the line $y = 2$, taken inside the parabola. [4]

SECTION C · Stretch

C1 The roots are the x-coordinates where line and curve meet. Two distinct roots mean two crossing points; a repeated root means the two intersections have merged into a single touch, which is tangency; no real roots mean the line misses the curve altogether. The discriminant therefore settles the geometry without any sketch. [3]