inkMaths

ANSWER BOOK

Tangents, turning points and curve behaviour

Differentiation · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $dy/dx = 2x$ gives gradient 4 at the point. Tangent: $y - 4 = 4(x - 2)$, so $y = 4x - 4$. Derivative for the slope, straight-line kit for the rest. [2]

A2 The normal is perpendicular to the tangent, so its gradient is the negative reciprocal, $-¼$. Then $y - 4 = -¼(x - 2)$, which tidies to $y = -x/4 + 9/2$. One derivative feeds both lines. [2]

SECTION B · Standard

B1 $dy/dx = 3x^2 - 6x - 9 = 3(x - 3)(x + 1)$, zero at $x = -1$ and $x = 3$. The second derivative $6x - 6$ is -12 at $x = -1$, a maximum, and $+12$ at $x = 3$, a minimum. The points are $(-1, 10)$ and $(3, -22)$: heights come from the original curve, never from the derivative. [4]

B2 The curve falls where dy/dx < 0, and the factorised derivative $3(x - 3)(x + 1)$ is negative between its roots: -1 < x < 3. Between the hilltop and the valley, the only way is down. [3]

B3 $dy/dx = 3x^2$, which is zero at $x = 0$, so the origin is stationary. But $3x^2$ is positive on both sides: the curve climbs into the point and climbs away from it. A flat moment on an uphill road: a point of inflection with a horizontal tangent, and the reason $f'' = 0$ alone classifies nothing. [3]

SECTION C · Stretch

C1 With one side x, the other is $20 - x$, and $A = x(20 - x) = 20x - x^2$. Then $dA/dx = 20 - 2x$, zero at $x = 10$, and d²A/dx² = -2 < 0 confirms a maximum. The rectangle is a 10 by 10 square with area 100 cm^2. Write the quantity as a function of one variable, then let the calculus do the choosing. [4]

C2 At a stationary point, f″ > 0 means a minimum and f″ < 0 a maximum. If $f'' = 0$ the test is silent, not negative: check the sign of f' on each side of the point, and let the gradient's story classify it. [2]