inkMaths

ANSWER BOOK

Taylor series

Further Pure 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $f(x) = f(a) + f'(a)(x - a) + f''(a)(x - a)^2/2! + \dots$: the coefficient of each power is the matching derivative at a, divided by the factorial of that power. [2]

A2 Every derivative of the exponential equals itself, so all the values at $x = 1$ are e. The series is $e + e(x - 1) + e(x - 1)^2/2 + \dots$, which is e times the standard exponential series shifted along. [3]

SECTION B · Standard

B1 Derivatives at 1: $\ln 1 = 0$, $1/x = 1$, $-1/x^2 = -1$, $2/x^3 = 2$. So $\ln x = (x - 1) - (x - 1)^2/2 + (x - 1)^3/3 + \dots$. At $x = 1.1$: $0.1 - 0.005 + 0.000333 = 0.09533$, against the true 0.09531. Three terms give four decimal places, because the anchor sits so close. [4]

B2 At $\pi/4$: $\tan = 1$, $\sec^2 = 2$, and the second derivative $2 \sec^2 x \tan x = 4$. So $\tan x = 1 + 2(x - \pi/4) + 2(x - \pi/4)^2 + \dots$. At the stated point: $1 + 0.2 + 0.02 = 1.22$, against the true 1.2230. Correct to 2 decimal places from a quadratic. [4]

B3 Every term after the first carries a power of $(x - a)$, which is small near the anchor, so the neglected terms are tiny there. A Maclaurin series carries powers of x instead, which are large when x is far from the origin, so far more terms are needed for the same accuracy. [3]

SECTION C · Stretch

C1 Derivatives at 9: $\sqrt{9} = 3$, $1/(2\sqrt{x}) = 1/6$, and $-1/(4x^{3/2}) = -1/108$. So $\sqrt{x} = 3 + (x - 9)/6 - (x - 9)^2/216 + \dots$. At $x = 9.6$: $3 + 0.1 - 0.001667 = 3.098$, against the true 3.0984. Anchoring at the nearest perfect square is what makes the arithmetic this light. [4]