inkMaths

ANSWER BOOK

Testing variances: chi-squared and the F-distribution

Further Statistics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The statistic is $(n - 1)S^2/\sigma^2$, where S^2 is the sample variance and σ^2 the value claimed under the null hypothesis. It follows a chi-squared distribution on $n - 1$ degrees of freedom, provided the population is normal. [2]

A2 H_0: $\sigma^2 = 4$; H_1: σ^2 > 4, one-tailed at 5% with 24 degrees of freedom. The statistic is 24 × 6.4/4 = 38.4. Since 38.4 > 36.415, reject H_0: there is evidence at the 5% level that the variance exceeds 4. [3]

SECTION B · Standard

B1 The numerator is $(n - 1)s^2 = 24 \times 6.4 = 153.6$. Dividing by the larger value gives the lower limit, 153.6/39.364 = 3.90, and dividing by the smaller gives the upper limit, 153.6/12.401 = 12.39. So the interval is (3.90, 12.39). The two ends look swapped because the statistic sits in the denominator. [4]

B2 H_0: $\sigma_1^2 = \sigma_2^2$; H_1: σ_1^2 > σ_2^2. Put the larger sample variance on top: F = 24.5/8.2 = 2.99 on 12 and 8 degrees of freedom, numerator first. Since 2.99 < 3.284, do not reject H_0: there is insufficient evidence at the 5% level that the first population is more variable, despite the sample variances differing by a factor of three. [4]

B3 With the larger variance on top the ratio is at least 1, so only the upper tail of the F table is ever needed and one column of critical values suffices. The degrees of freedom must follow the variances: numerator degrees of freedom first, denominator second. Swapping them gives a different critical value, since the F distribution is not symmetric in its two parameters. [3]

SECTION C · Stretch

C1 H_0: $\sigma^2 = 5$; H_1: $\sigma^2 \neq 5$, with 15 degrees of freedom and 2.5% in each tail. The statistic is 15 × 2.5/5 = 7.5. Since 6.262 < 7.5 < 27.488 the result lies between the two critical values: do not reject H_0. Note how far apart the two values are: the chi-squared distribution is strongly skewed, so the region is nothing like symmetric about the expected value of 15. [4]