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ANSWER BOOK

The binomial expansion

Sequences and series · Year 12-13

7 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $8!/(3! \times 5!) = (8 \times 7 \times 6)/(3 \times 2 \times 1) = 336/6 = 56$. Only three factors of the top survive the cancelling; computing 8! in full is the long way round. [2]

A2 1, 5, 10, 10, 5, 1: six entries for six terms. Each entry is the sum of the two above it, and the row reads the same in both directions. [2]

SECTION B · Standard

B1 Row 4 gives 1, 4, 6, 4, 1. Building each term: 81, then $4 \times 27 \times 2x = 216x$, then $6 \times 9 \times 4x^2 = 216x^2$, then $4 \times 3 \times 8x^3 = 96x^3$, then $16x^4$. So $(3 + 2x)^4 = 81 + 216x + 216x^2 + 96x^3 + 16x^4$. At $x = 1$ the bracket is $5^4 = 625$, and $81 + 216 + 216 + 96 + 16 = 625$: the cheap check passes. [4]

B2 $1 + {}^8C_1(3x) + {}^8C_2(3x)^2 = 1 + 24x + 28 \times 9x^2 = 1 + 24x + 252x^2$. The replacement $3x$ goes into the formula whole, and gets squared whole: forgetting to square the 3 is the slip to guard against. [3]

B3 The terms are $2^5 = 32$, then $5 \times 2^4x = 80x$, then $10 \times 2^3x^2 = 80x^2$. Setting $x = 0.002$: $32 + 0.16 + 0.00032 = 32.16032$. The true value is 32.1603203…, so the truncation is already good to six decimal places; the discarded x^3 term is only 40×0.002^3, three parts in ten million. [6]

SECTION C · Stretch

C1 The terms are 1, then $10(x/4) = 2.5x$, then $45(x/4)^2 = 2.8125x^2$. Setting $x/4 = 0.0025$ means $x = 0.01$, giving $1 + 0.025 + 0.00028125 = 1.02528125$. The true value is 1.0252832…, matching to five decimal places: small x makes the tail negligible. [5]

C2 The coefficient ${}^nC_r$ counts the ways of choosing which r brackets supply the second letter; choosing those r is the same act as choosing the other $n - r$ to supply the first. So ${}^nC_r = {}^nC_{n-r}$, and the row is a palindrome. [2]