inkMaths

ANSWER BOOK

The Central Limit Theorem

Further Statistics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 For a population with mean μ and variance σ^2, the mean of a large independent sample is approximately $N(\mu, \sigma^2/n)$, whatever the parent's shape. The approximation improves as n grows. [2]

A2 Approximately $N(75, 256/64) = N(75, 4)$. The standard error is $\sqrt{4} = 2$, which is also $16/\sqrt{64}$. Dividing the variance by n, not the standard deviation, is the step that decides the answer. [3]

SECTION B · Standard

B1 z = (78 - 75)/2 = 1.5, so P(mean > 78) = P(Z > 1.5) = 0.0668. The parent's shape never entered the working, which is exactly what the theorem buys. [4]

B2 A Poisson has mean and variance both 9, so the sample mean is approximately $N(9, 9/36) = N(9, 0.25)$, standard error 0.5. Then $z = (8.5 - 9)/0.5 = -1$, giving $P \approx 0.1587$. A discrete, skewed parent is no obstacle at all. [4]

B3 $6/\sqrt{n} \le 0.5$ gives $\sqrt{n} \ge 12$, so $n \ge 144$. Halving that requirement to 0.25 would need $n \ge 576$: four times the data for half the error, because the root is what does the work. [3]

SECTION C · Stretch

C1 Individual observations keep the parent's distribution no matter how many are collected; taking more of them changes nothing about any single one. What becomes approximately normal is the distribution of the sample mean, taken over repeated samples. The theorem is about a statistic, not about the raw data. [4]