inkMaths

ANSWER BOOK

The continuous uniform distribution

Further Statistics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 f(x) = 1/(b - a) for a ≤ x ≤ b and zero elsewhere. F(x) = 0 for x < a, (x - a)/(b - a) for a ≤ x ≤ b, and 1 for x > b. Both branches outside the interval are part of the answer. [2]

A2 Mean = (5 + 17)/2 = 11. Variance = $(17 - 5)^2/12$ = 144/12 = 12. Standard deviation = √12 = 3.46 to three significant figures, which is about 29% of the range as always. [3]

SECTION B · Standard

B1 The height is 1/12. P(X < 8) = 3/12 = 0.25. P(9 < X < 15) = 6/12 = 0.5. For the conditional probability, P(X > 14 and X > 11) = P(X > 14) = 3/12, and P(X > 11) = 6/12, so the answer is (3/12)/(6/12) = 0.5. [4]

B2 The error is equally likely anywhere in the rounding interval, so it follows U(−5, 5) with mean 0. Variance = $10^2/12$ = 8.333, so the standard deviation is 2.89. An error above 4 in size covers two stretches of length 1 out of a range of 10, so the probability is 2/10 = 0.2. [4]

B3 The wait follows U(0, 15), so the expected wait is the midpoint, 7.5 minutes. P(wait > 10) = 5/15 = 1/3. The model assumes arrival independent of the timetable, which fails for a passenger who has checked it. [3]

SECTION C · Stretch

C1 F(x) = ∫ 1/(b - a) dt from a to x = (x - a)/(b - a) on the interval, which is 0 at a and 1 at b as required. E(X) = ∫ x/(b - a) dx from a to b = $(b^2 - a^2)/[2(b - a)]$. Factorising the difference of squares gives (b + a)(b - a)/[2(b - a)] = (a + b)/2, the midpoint, which the symmetry of the rectangle would have predicted. [4]