inkMaths

ANSWER BOOK

The derivative from first principles

Differentiation · Year 12–13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $f'(x) = \lim$ as $h \to 0$ of $[f(x + h) - f(x)]/h$. The fraction is the gradient of a chord from x to $x + h$; the limit slides the far end of the chord into the near end, turning the chord into the tangent. [2]

A2 With $h = 0.1$: $(3.1^2 - 9)/0.1 = 6.1$. With $h = 0.01$: 6.01. The chords are settling towards 6, which the derivative $2x = 6$ confirms. Numerical chords are evidence; the limit is the proof. [3]

SECTION B · Standard

B1 The chord gradient is $[(x + h)^2 - x^2]/h = (2xh + h^2)/h$. For $h \neq 0$ this divides to $2x + h$, and as $h \to 0$ the surviving part is $2x$. The three moves are expand, divide, let h vanish, and the middle one needs $h \neq 0$ said aloud. [4]

B2 The chord gradient is $[3(x + h)^2 + 2(x + h) - 3x^2 - 2x]/h = (6xh + 3h^2 + 2h)/h = 6x + 3h + 2$ for $h \neq 0$. As $h \to 0$ this becomes $6x + 2$. Each term contributes its own derivative, which is the termwise rule being born. [4]

B3 The chord gradient from $x = 2$ is $[(2 + h)^3 - 8]/h = (12h + 6h^2 + h^3)/h = 12 + 6h + h^2$. As $h \to 0$ the gradient is 12. Working at the specific point keeps the binomial expansion short: only the 8 and the h terms appear. [3]

SECTION C · Stretch

C1 $f'(x) = 3x^2 - 12x$ and $f''(x) = 6x - 12$, which vanishes at $x = 2$. The second derivative is the derivative of the gradient function: it measures how fast the slope itself is changing, and its zero marks where the curve switches from bending one way to bending the other. [4]