inkMaths

ANSWER BOOK

The Euclidean algorithm and Bezout's identity

Further Pure 2 · Year FM

6 questions · 21 marks

NAVY EDITION

SECTION A · Warm-up

A1 1001 = 2 × 357 + 287; 357 = 1 × 287 + 70; 287 = 4 × 70 + 7; 70 = 10 × 7 + 0. The last non-zero remainder is 7. No factorisation of either number was needed. [3]

A2 From the third line, 7 = 287 - 4 × 70. Substituting 70 = 357 - 287 gives 7 = 5 × 287 - 4 × 357, and substituting 287 = 1001 - 2 × 357 gives 7 = 5 × 1001 - 14 × 357. So x = 5, y = -14, and checking: 5005 - 4998 = 7. [3]

SECTION B · Standard

B1 2024 = 2 × 748 + 528; 748 = 1 × 528 + 220; 528 = 2 × 220 + 88; 220 = 2 × 88 + 44; 88 = 2 × 44 + 0, so the factor is 44. Unwinding: 44 = 220 - 2 × 88 = 5 × 220 - 2 × 528 = 5 × 748 - 7 × 528 = 19 × 748 - 7 × 2024. Check: 14212 - 14168 = 44. [4]

B2 391 = 1 × 299 + 92; 299 = 3 × 92 + 23; 92 = 4 × 23 + 0, so the highest common factor is 23 and the numbers are not coprime. (Indeed 391 = 17 × 23 and 299 = 13 × 23, though the algorithm found the factor without looking for it.) [3]

B3 26 = 2 × 11 + 4; 11 = 2 × 4 + 3; 4 = 1 × 3 + 1, so the numbers are coprime. Unwinding: 1 = 4 - 3 = 3 × 4 - 11 = 3 × 26 - 7 × 11. Reducing modulo 26 gives -7 × 11 ≡ 1, so the inverse is -7 ≡ 19. Checking: 11 × 19 = 209 = 8 × 26 + 1. [4]

SECTION C · Stretch

C1 The highest common factor of 12 and 45 is 3, which divides 6, so solutions exist. Euclid gives 45 = 3 × 12 + 9 and 12 = 1 × 9 + 3, so 3 = 12 - 9 = 4 × 12 - 45. Doubling: 6 = 8 × 12 - 2 × 45, giving x = 8, y = -2. Adding multiples of 45/3 = 15 to x and subtracting the matching multiples of 12/3 = 4 from y gives the general solution x = 8 + 15t, y = -2 - 4t for integer t. [4]