inkMaths

ANSWER BOOK

The general binomial expansion

Sequences and series · Year 12–13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 For whole n the coefficient n(n - 1)…(n - r + 1)/r! eventually picks up the factor zero and every later term dies. For n = -1 the factors -1, -2, -3, … never hit zero, so the series runs for ever, and it only converges for |x| < 1. [2]

A2 With n = -1: 1 - x + x^2 - x^3 + …, valid for |x| < 1. The alternating signs come from the coefficient factors -1, -2, -3 dividing by 1!, 2!, 3!: each ratio is ±1. This is the geometric series 1/(1 + x) in binomial clothing. [3]

SECTION B · Standard

B1 Apply the formula with n = ½ and x replaced by 4x: 1 + ½(4x) + [½ × (-½)/2]$(4x)^2$ = 1 + 2x - $2x^2$. Validity needs |4x| < 1, so |x| < ¼. The replacement goes in whole, and the validity window shrinks by the same factor of 4. [4]

B2 With n = -2 and 2x in place of x: 1 + (-2)(2x) + [(-2)(-3)/2]$(2x)^2$ = 1 - 4x + $12x^2$, valid for |x| < ½. Setting x = 0.01: 1 - 0.04 + 0.0012 = 0.9612, against a true value of 0.961169…: agreement to four decimal places from three terms. [4]

B3 The bracket does not start with 1, so factor the 4 out: $(4 + x)^{1/2}$ = $2(1 + x/4)^{1/2}$. Expanding the inner bracket: 1 + x/8 - x^2/128 + …, and doubling gives 2 + x/4 - x^2/64. Validity needs |x/4| < 1, so |x| < 4. Forgetting that the factored-out 4 comes through as $4^{1/2}$ = 2, not 4, is where this question usually goes wrong. [4]

SECTION C · Stretch

C1 Set x = 0.2: 2 + 0.05 - 0.04/64 = 2.049375, against a true value of 2.0493901…: four decimal places from three terms. The accuracy comes from the effective variable x/4 = 0.05 being far inside the validity window, so each term is roughly twenty times smaller than the one before. [3]